# Dithered Sliding Mode Control for Switched Systems

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***Abstract*— Dithering is proposed as a technique for the regulation of the boundary layer for switched systems under sliding mode control. It is proved that dithering maintains the state of the system close to the ideal sliding manifold and that state error and practical stability depend on the dither amplitude, period and shape, all by preserving the natural switching mode of the system. Experiments on a DC/DC buck power converter confirm the theoretical results.**

***Index Terms*— Dither techniques, power electronics, sliding mode, switched systems.**

## I. Introduction

Practical applications of ideal sliding mode control are typically affected by chattering [1]. A widely used solution to the chattering problem is based on the boundary layer idea [2], [3]. The boundary layer controller consists of a smooth interpolation of the relay characteristic typical of the sliding controller (resulting for instance in a saturation or a sigmoid) and its design should achieve a good compromise between robustness, which asks for increasing the controller gain, and chattering mitigation, which needs a reduction of the controller gain [1], [4]. Despite its common use, the smoothness of the boundary layer controller makes this solution not natural for systems that by their nature operate in switching (i.e. on-off) mode.

In this paper it is proved that dithering can be used as a boundary layer approach for switched systems with sliding mode control. A dithered system is a system with nonsmooth nonlinearities (such as relays) in which a suitable high frequency signal (the *dither*) is injected at the input of the discontinuous nonlinearity. Dithered systems can be analytically approximated by an averaged system, without dither, in which the discontinuous nonlinearity is replaced by an averaged nonlinearity whose shape depends on the dither signal waveform [5], [6]. In this paper it is shown that the boundary layer can be adapted by simply changing the dither shape and the error between the state of the dithered system and the state of the boundary layer (the averaged) system is of order of the dither period.

An interesting class of switched systems for which sliding mode control is widely used is represented by power converters [7], [8], [9]. The theoretical results here proposed are experimentally verified on a sliding mode controlled DC/DC converter for which the problem of chattering mitigation is typically solved through a suitable use of hysteretic controllers [10], [11]. Simulation results were presented by the authors in [12]. The experiments reported in the paper confirm the use of dither as an effective alternative solution for the implementation of a boundary layer controller.



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## II. Preliminaries

Consider the nonlinear system

$$\dot{\eta} = f(\eta, \xi) \tag{1a}$$
$$\dot{\xi} = f_a(\eta, \xi) + G_a(\eta, \xi)u \tag{1b}$$

where $\eta \in \mathbb{R}^{n-p}$, $\xi \in \mathbb{R}^p$, $u \in \mathbb{R}^p$ is the input of the system, $G_a(\eta, \xi)$ is non singular and $f(\eta, \xi)$ and $f_a(\eta, \xi)$ are globally Lipschitz functions. From (1) the *sliding mode controlled system* can be represented through the following model [2]:

$$\dot{\eta} = f(\eta, \xi) \tag{2a}$$
$$\dot{\xi} = f_0(\eta, \xi) - \alpha(\eta, \xi)w \tag{2b}$$

where $\alpha(\eta, \xi)$ is a scalar function satisfying the condition $\alpha(\eta, \xi) \geq \alpha_0 > 0$, $f_0$ is a globally Lipschitz function obtained from the control law

$$u = G_a^{-1}[-f_a(\eta, \xi) + \frac{\partial \phi(\eta)}{\partial \eta} f(\eta, \xi)] + G_a^{-1} v \tag{3a}$$
$$v = -\alpha(\eta, \xi)w \tag{3b}$$

with $\phi(\eta)$ a Lipschitz function that stabilizes the system $\dot{\eta} = f(\eta, \phi(\eta))$ and the control variable $w \in \mathbb{R}^p$ is given by the component-wise relay characteristic:

$$w_i = n(\sigma_i) = \mathrm{sgn}(\sigma_i) \triangleq \begin{cases} +1, & \sigma_i > 0 \\ -1, & \sigma_i < 0 \end{cases} \tag{4}$$

for $i = 1, \ldots, p$, with $\sigma(\eta, \xi) = \xi - \phi(\eta)$ and $\sigma = 0$ represents the sliding manifold.

The boundary layer idea is based on the use of a continuous approximation of the discontinuous sliding mode controller. A typical choice consists of replacing the relay characteristic (4) by a saturation function, i.e. by using in (2)

$$w_i = \mathrm{sat}\left(\frac{\sigma_i}{\epsilon}\right) \tag{5}$$

for $i = 1, \ldots, p$, where $1/\epsilon$ is the slope of the linear part of the saturation. The resulting system is the so-called boundary layer system.

The *dithered system* corresponding to (2) is defined as

$$\begin{aligned} \dot{\eta}_d &= f(\eta_d, \xi_d) && (6a) \\ \dot{\xi}_d &= f_0(\eta_d, \xi_d) - \alpha(\eta_d, \xi_d)w_d && (6b) \\ w_d &= n(\sigma(\eta_d, \xi_d) + \delta) && (6c) \\ \sigma(\eta_d, \xi_d) &= \xi_d - \phi(\eta_d) && (6d) \end{aligned}$$

where the components of the dither signal $\delta$ are assumed to be periodic of period $T$ with zero mean value and the relay characteristic $n$ is defined in (4). Note that the function $\sigma$ is Lipschitz with respect to its arguments. When the dither is a sawtooth high frequency signal, (6) can be interpreted as a pulse width modulated system with $\sigma$ being the modulating signal. In our analysis, instead, the shape of $\delta$ is chosen depending on the desired equivalent behaviour of the dithered system represented by a corresponding *averaged system*, defined as

$$\dot{\eta}_s = f(\eta_s, \xi_s) \tag{7a}$$
$$\dot{\xi}_s = f_0(\eta_s, \xi_s) - \alpha(\eta_s, \xi_s) w_s \tag{7b}$$
$$w_s = N(\sigma(\eta_s, \xi_s)) \tag{7c}$$
$$\sigma(\eta_s, \xi_s) = \xi_s - \phi(\eta_s) \tag{7d}$$

where $N(z)$ is the so-called averaged nonlinearity. The averaged (or smoothed) nonlinearity is obtained by evaluating the time averaging of the output of the relay on a dither period $T$ by assuming $z$ constant:

$$N(z) = \frac{1}{T}\int_0^T n(z + \delta(t))\mathrm{d}t. \tag{8}$$

The function $N(z)$ depends on the signal $\delta$ and its shape. If the dither signal is a sawtooth (or a triangular) waveform which varies between $-M_\delta$ and $+M_\delta$ and period $T$, it is easy to show (by applying (8)) that

$$N(z) = \mathrm{sat}(z/M_\delta) = \begin{cases} -1, & z < -M_\delta \\ z/M_\delta, & |z| \le M_\delta \\ +1, & z > M_\delta. \end{cases} \tag{9}$$

Note that $N(z)$ is Lipschitz (with Lipschitz constant equal to $1/M_\delta$) while the original relay nonlinearity was discontinuous: dither has "smoothed" the relay. Other averaged nonlinearities can be obtained by considering different dither signals [6]. For instance, if a sinusoidal dither of amplitude $M_\delta$ is used, the corresponding averaged nonlinearity is

$$N(z) = \begin{cases} -1 & z < -M_\delta \\ \dfrac{2}{\pi}\sin^{-1}\dfrac{z}{M_\delta} & |z| \le M_\delta \\ 1 & z > M_\delta \end{cases} \tag{10}$$

which is absolutely continuous.

## III. Dithered sliding mode controller

### A. Attractiveness

The state of the dithered system will reach in finite time a suitable neighborhood of the sliding manifold $\sigma(\eta, \xi) = 0$, say $\Omega$, which depends on the dither amplitude. That is proved in the following lemma.

***Lemma 1:*** Consider the dithered system (6) in which $\delta$ is any periodic waveform of amplitude $M_\delta$. For any initial condition $(\eta_{d0}, \xi_{d0}) = (\eta_0, \xi_0) \in \mathbb{R}^n$, the trajectories of the dithered system reach the region

$$\Omega = \{(\eta_d, \xi_d) : |\sigma_i(\eta_d, \xi_d)| \le M_\delta, i = 1, \ldots, p\} \tag{11}$$

after a finite time $\Delta$.

*Proof:* Outside the region $\Omega$, the dither signal $\delta$ has no effect since when $|\sigma_i| > M_\delta$ the corresponding argument of $n$ in (6) does not change signum. So the dithered system is equal to the system (2). Therefore, outside $\Omega$, from the definition of the sliding manifold one can write:

$$\begin{aligned} \dot{\sigma}(\eta_d, \xi_d) &\equiv \dot{\sigma}(\eta, \xi) = \dot{\xi} - \frac{\partial \phi(\eta)}{\partial \eta}\dot{\eta} = \\ &= f_0(\eta, \xi) - \alpha(\eta, \xi) w - \frac{\partial \phi(\eta)}{\partial \eta} f(\eta, \xi) = \\ &= -\alpha(\eta, \xi) w = -\alpha(\eta, \xi) n(\sigma(\eta, \xi)). \end{aligned} \tag{12}$$

Now, define $V = \frac{1}{2}\sum_{i=1}^{p} \sigma_i^2$ as a Lyapunov function candidate for the system (12). Considering the $i$-th component, it follows that:

$$\dot{V}_i = \sigma_i \dot{\sigma}_i = -\sigma_i \alpha(\eta, \xi) n(\sigma). \tag{13}$$

If $|\sigma_i| > M_\delta$ (that is outside $\Omega$), then:

$$\dot{V}_i = -\sigma_i \alpha(\eta, \xi) n(\sigma_i) = -|\sigma_i| \alpha(\eta, \xi) \le -\alpha_0 |\sigma_i|. \tag{14}$$

So, the inequality $\dot{V}_i \le -\alpha_0 |\sigma_i|$ ensures that the trajectory reaches the region (11) in a finite time. ■

### B. Stability of the averaged system

It is now possible to show that over an infinite time horizon the state of the dithered system will remain close to the state of the averaged system when the latter tends asymptotically to the origin. Note that this result is valid everywhere and not only outside $\Omega$ where it is obvious because the sliding mode system (2), the dithered system (6) and the averaged system (7) are the same. As a consequence of this result one can conclude that the state of the dithered system will remain about the sliding manifold over an infinite time horizon.

***Lemma 2:*** Consider the averaged system (7). Suppose that the nonlinearity $N(z)$ can be written as $N(z) = \overline{N}(z/M_\delta)$ that satisfies the condition

$$z\overline{N}(z) \ge \gamma z^2, \qquad \forall |z| \le 1 \tag{15}$$

with $\gamma > 0$. So, if the origin of the system $\dot{\eta}_s = f(\eta_s, \phi(\eta_s))$ is globally exponentially stable, then there exist $M_\delta^* > 0$ such that for all $0 < M_\delta < M_\delta^*$ the origin of the averaged system is globally uniformly asymptotically stable.

*Proof:* From (7d) it follows that:

$$\dot{\sigma} = \dot{\xi}_s - \frac{\partial \phi(\eta_s)}{\partial \eta_s}\dot{\eta}_s = -\alpha(\eta_s, \xi_s) N(\sigma). \tag{16}$$

Since outside the boundary layer the dithered and averaged systems are equivalent, by following the same proof of the Lemma 1, it can be showed that every trajectory of the system reach the set $\Omega$ in a finite time. Inside the region $\Omega$, the dynamic model of the system is given by:

$$\dot{\eta}_s = f(\eta_s, \sigma + \phi(\eta_s)) \tag{17a}$$
$$\dot{\sigma} = -\alpha(\eta_s, \sigma + \phi(\eta_s)) N(\sigma). \tag{17b}$$

By the converse Lyapunov theorem [2], there exists a radially unbounded Lyapunov function $V_0(\eta_s)$ that satisfies

$$\begin{aligned} k_1|\eta_s|^2 \leq V_0(\eta_s) &\leq k_2|\eta_s|^2 \\ \frac{\partial V_0}{\partial \eta_s} f(\eta_s, \phi(\eta_s)) &\leq -k_3|\eta_s|^2 \\ \left|\frac{\partial V_0}{\partial \eta_s}\right| &\leq k_4|\eta_s|. \end{aligned}$$

Since $f$ is globally Lipschitz, one can say that:

$$|f(\eta_s, \phi(\eta_s) + \sigma) - f(\eta_s, \phi(\eta_s))| \leq L|\sigma|. \quad (18)$$

Now, by choosing as a candidate Lyapunov function

$$W = V_0(\eta_s) + \frac{1}{2}\sum_{i=1}^{p} \sigma_i^2 \quad (19)$$

taking the derivative of (19) leads to:

$$\begin{aligned} \dot{W} &= \frac{\partial V_0}{\partial \eta_s} f(\eta_s, \sigma + \phi(\eta_s)) + \sum_{i=1}^{p} \sigma_i \dot{\sigma}_i \\ &= \frac{\partial V_0}{\partial \eta_s}[f(\eta_s, \sigma + \phi(\eta_s)) - f(\eta_s, \phi(\eta_s))] \\ &\quad + \frac{\partial V_0}{\partial \eta_s} f(\eta_s, \phi(\eta_s)) + \sum_{i=1}^{p} \sigma_i \dot{\sigma}_i \\ &\leq k_4 L|\eta_s||\sigma| - k_3|\eta_s|^2 - \alpha_0 \sum_{i=1}^{p} N(\sigma_i)\sigma_i. \quad (20) \end{aligned}$$

Since inside $\Omega$ it holds $|\sigma_i| \leq M_\delta$, the condition (15) is valid for $z = \sigma_i/M_\delta$ and one can write:

$$\dot{W} \leq k_4 L|\eta_s||\sigma| - k_3|\eta_s|^2 - \frac{\alpha_0}{M_\delta}\gamma|\sigma|^2. \quad (21)$$

By choosing $M_\delta < 4\frac{k_3\alpha_0\gamma}{k_4^2 L^2} = M_\delta{}^*$, the right hand side of (21) can be made negative. ■

In [2] and [13] results similar to Lemma 2 have been presented. Assuming that disturbances are not present, the assumptions on the nonlinearity in [2] and [13] are more restrictive than (15). Indeed, the result in [2, Exercise 14.11] is equivalent to assume $\gamma = \overline{N}(1)$ and therefore nonlinearity (10) could not be considered. In [13] constraints on the nonlinearity $N$ seem to be even more restrictive, for instance $N(z) = \frac{2}{\pi}\arctan(\frac{z}{M_\delta})$, which satisfies (15), violates such constraints. Note that choosing $\delta(t)$ Lipschitz at the zero crossing time instants implies that (15) is verified, i.e. $\overline{N}$ has non zero slope in the origin.

### C. Boundary layer through dithering

The following lemma shows that if the dithered system (6) has no sliding, the error between the state of the dithered system and the state of the averaged system is of order of the dither period.

***Lemma** 3:* Consider the dithered system (6) and the averaged system (7) with the assumptions that the dithered system has an absolutely continuous solution and that the dither $\delta$ is $T$-periodic and such that the averaged nonlinearity $N$ is Lipschitz continuous. Then the averaged system (7) has a unique absolutely continuous solution on $[0, \infty)$. Moreover, for any given $\Delta > 0$ and initial condition $x_0$, it holds that

$$|x_d(t, x_0) - x_s(t, x_0)| = O(T), \quad \forall t \in [0, \Delta] \quad (22)$$

where

$$x_d = \begin{pmatrix} \eta_d \\ \xi_d \end{pmatrix}, \quad x_s = \begin{pmatrix} \eta_s \\ \xi_s \end{pmatrix}.$$

*Proof:* The result can be obtained as a particular case of the averaging theorem proved in [6]. ■

***Lemma** 4:* Consider the averaged system (7) and the dithered system (6). Suppose that the result of Lemma 3 holds and the origin of the state space of the averaged system (7) is globally uniformly asymptotically stable. Then the origin of the state space of the dithered system (6) is practically globally uniformly asymptotically stable.

*Proof:* The result can be obtained as a particular case of the theorem proved in [14]. ■

It is now possible to prove the main result of the paper.

***Theorem** 1:* Consider the dithered system (6) and the averaged system (7) in which the nonlinearity satisfies the condition (15). Suppose that the origin of the system $\dot{\eta}_s = f(\eta_s, \phi(\eta_s))$ is globally exponentially stable and the hypotheses of Lemma 4 hold. Then, for any initial condition $x_0$, it holds that

$$|x_d(t, x_0) - x_s(t, x_0)| = O(T), \quad \forall t \in [0, \infty). \quad (23)$$

*Proof:* Applying Lemma 2 it follows that the origin of the averaged system is globally uniformly asymptotically stable. So, in force of Lemmas 3 and 4 the condition (22) holds over an infinite time horizon. ■

The above theorem allows to conclude that the dithered sliding controller will maintain the state close to the sliding manifold.

### D. Dither design

After that the sliding mode controller has been designed, the practical implementation consists in the choice of the averaged nonlinearity of the boundary layer system (7) in order to satisfy some desired stability or robustness performances. Such averaged nonlinearity corresponds to a particular dither waveform (see (8)). As an example, if the averaged nonlinearity is a saturation (with a slope of $1/M_\delta$ in the linear region), the corresponding dither can be a sawtooth or a triangular waveform of amplitude $M_\delta$. The robustness of the closed loop system can be improved by decreasing $M_\delta$ since the gain of the saturation increases and approximates with better accuracy the relay characteristic. If a sinusoidal dither is used, the averaged nonlinearity (10) corresponds to a sort of variable feedback gain. In fact, when the state is close to the sliding manifold the controller gain is “small” and thus the controller effect is weak, whereas the effect of the controller increases when the state goes away from the sliding manifold. Again performance of the averaged system and accuracy with respect to the ideal sliding mode controlled system can be varied by changing the dither amplitude and period, respectively. The dither will provide a constant switching frequency that can be varied by changing the dither period, the state will remain close to the sliding manifold according to (23), and the local attractiveness of the sliding manifold can be varied by changing the dither shape and its amplitude. Moreover, from a global point of view (i.e. large deviations) the dithered controller provides the same performance of the ideal sliding mode one. In that sense, the proposed solution is different from the classical Pulse Width Modulation technique for which local averaged system are typically used for the controller design.

## IV. Experiments on a DC/DC Buck Converter

Consider the DC/DC buck converter represented in Figure 1. A possible sliding mode controlled converter model can be represented as follows

$$\dot{\eta}_1 = \eta_2 - V_{\text{ref}} \tag{24a}$$

$$\dot{\eta}_2 = -\frac{1}{RC}\eta_2 + \frac{1}{C}\xi \tag{24b}$$

$$\dot{\xi} = -\frac{1}{L}\eta_2 + \frac{E}{2L} - \frac{E}{2L}n(\sigma) \tag{24c}$$

$$\sigma = k_1\xi + k_2\eta_1 \tag{24d}$$

which can be rewritten in the form (2). It is simple to show that the system satisfies the sliding condition and reaches the sliding manifold in a finite time. When the state lies on the sliding manifold $\sigma = 0$ the dynamics of the system are governed by (24a) and (24b). Thus by replacing $\xi$ (obtained from the relation $\sigma \equiv 0$) in (24b), the equilibrium point of the controlled system is given by $[-\frac{V_{\text{ref}}k_1}{Rk_2}, V_{\text{ref}}, \frac{V_{\text{ref}}}{R}]'$. Furthermore, $k_1$ and $k_2$ control the convergence rate to the equilibrium point.

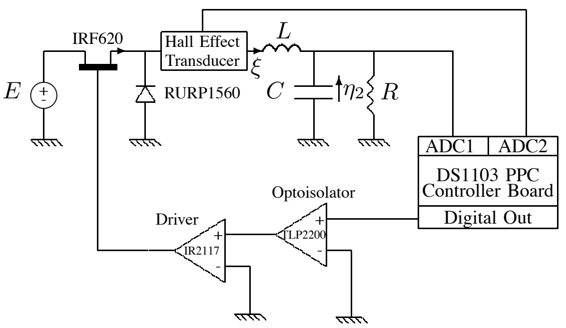


Fig. 1. Controlled DC/DC buck converter.

The values of the converter parameters are $E = 10$V, $V_{\text{ref}} = 6$V, $C = 220\mu$F, $L = 1$mH, $R = 8.9\Omega$, and the controller parameters are $k_1 = 0.5$ and $k_2 = 10$. The sliding controller is implemented through a dSPACE$^{\text{TM}}$ DS1103 PPC controller board with a sampling period of $10\mu$s. In practical implementation the digital controller can be replaced by inexpensive hardware components consisting of operational amplifiers (see (24d)) and a signal generator (for the dither). First let us verify if the use of a certain dither signal allows to obtain the theoretically predicted averaged behaviour. By evaluating the time averaging (on a dither period) of the sliding surface versus the output of the relay characteristic (duty cycle) during different dynamic operating conditions due to load and input changes, Figure 2 is obtained. The computed

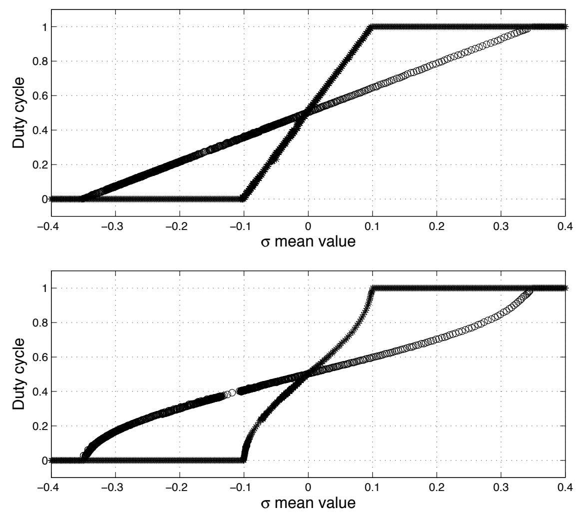


Fig. 2. Equivalent computed nonlinearity (with $T = 200\mu s$): (a) triangular dither (top diagram), (b) sinusoidal dither (bottom diagram) both with $M_\delta = 0.1, 0.35$.

averaged nonlinearities reproduce their analytical prediction obtained through (8). Figure 2 confirms that during transient the equivalence between the dithered and averaged system is preserved.

We can now look for some more theoretical insights by considering the experimental results obtained by varying the input voltage (a step from 10V to 14V) and the load (an abrupt change from 8.9Ω to 15Ω) using dither signals with different shapes and amplitudes. Figure 3 reports the corresponding results obtained with two different dithered systems. Other experimental results are synthesized in Table I. For comparative purposes, the usual power electronics approach to the boundary layer implementation, i.e. to use an hysteresis band [10] is also considered.

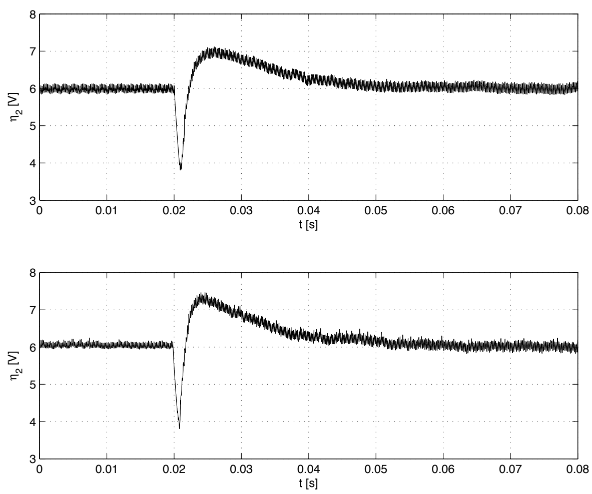


Fig. 3. Dynamic response of the output voltage to an input voltage variation (from 10V to 14V) for: (a) triangular dither (top diagram) and (b) sinusoidal dither (bottom diagram) both with $M_\delta = 0.25$ and $T = 200\mu s$.

TABLE I
EXPERIMENTAL RESULTS FOR INPUT VOLTAGE CHANGE.

| | Amplitude | Settling Time | Switching Frequency | Overshoot | Voltage Ripple |
|---|---|---|---|---|---|
| Hysteresis | $\pm 0.01$ | 20ms | 14kHz | 6.7% | 0.12V |
| | $\pm 0.1$ | 22ms | 5.4kHz | 8.3% | 0.18V |
| | $\pm 0.2$ | 25ms | 3.6kHz | 12% | 0.35V |
| Triangular dither | 0.1 | 21ms | 5kHz | 12.5% | 0.17V |
| | 0.25 | 30ms | 5kHz | 16.6% | 0.22V |
| | 0.35 | 38ms | 5kHz | 20% | 0.25V |
| Sinusoidal dither | 0.1 | 24ms | 5kHz | 15% | 0.22V |
| | 0.25 | 34ms | 5kHz | 21.7% | 0.25V |
| | 0.35 | 52ms | 5kHz | 26.7% | 0.25V |

TABLE II
EXPERIMENTAL RESULTS FOR LOAD CHANGE.

| | Amplitude | Settling Time | Switching Frequency | Overshoot | Voltage Ripple |
|---|---|---|---|---|---|
| Hysteresis | $\pm 0.01$ | 22ms | 14kHz | 41.7% | 0.18V |
| | $\pm 0.1$ | 25ms | 5.4kHz | 40% | 0.22V |
| | $\pm 0.2$ | 29ms | 3.6kHz | 36.7% | 0.3V |
| Triangular dither | 0.1 | 24ms | 5kHz | 35% | 0.21V |
| | 0.25 | 26ms | 5kHz | 25% | 0.24V |
| | 0.35 | 34ms | 5kHz | 21.7% | 0.25V |
| Sinusoidal dither | 0.1 | 28ms | 5kHz | 33.3% | 0.22V |
| | 0.25 | 35ms | 5kHz | 21.7% | 0.24V |
| | 0.35 | 49ms | 5kHz | 20% | 0.25V |

By increasing the amplitude of the triangular dither, the equivalent gain of the averaged nonlinearity decreases and the dynamic response is slower (see the settling time in Table I). In the case of a sinusoidal dither, experiments show a different behaviour (see Figure 3): now the dither ensures a fast transient only when the state is far from the sliding manifold. In fact, the sinusoidal dither allows a better rejection of disturbance due to the larger gain far from the sliding manifold. On the other hand, in a neighborhood of the sliding manifold the convergence is slower due to the smaller gain of the $\arcsin$ nonlinearity. It is clear that if the amplitude of the sinusoidal dither is smaller, the equivalent nonlinearity approaches the relay characteristic and the dynamics are faster. Note that the switching frequency is constant for each dither amplitude differently from the hysteresis case in which it changes by varying the band (see Table I).

Figure 4 shows the trajectories in the state space. Note that the ripple about the sliding manifold remains bounded by $\pm M_\delta$, so as predicted by Lemma 1.

Table II reports the experimental results obtained by varying the load. By computing from (24) the corresponding averaged model (which in this case is linear) it is simple to verify that the experiments confirm that the dithered system represents a good approximation of the boundary layer one.

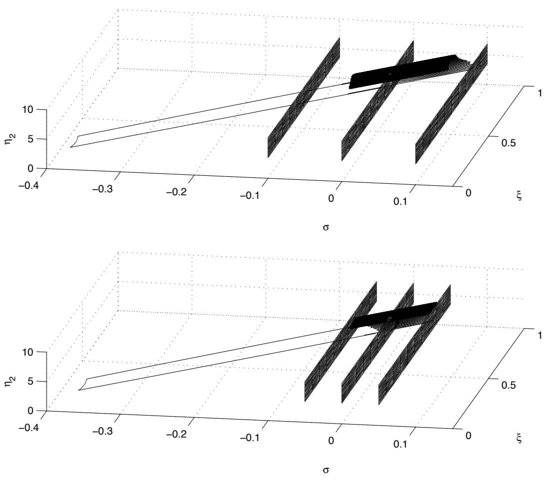


Fig. 4. Trajectory of the dithered system in the phase plane to an input variation (a change from 10V to 14V) for a triangular dither with $T = 100\mu s$ and $M_\delta = 0.1$ (top diagram) and $M_\delta = 0.05$ (bottom diagram).

## V. CONCLUSIONS

Dithering has been proved to be an efficient solution for the practical implementation of boundary layer in sliding mode controlled switched systems. By preserving the natural switching operating mode of the system, the proposed technique allows to adapt the boundary layer by simply changing the dither shape. Experiments obtained by considering a sliding mode controlled DC/DC buck power electronic converter validate the proposed technique. The experimental results confirm the converter behaviour predicted by the averaged analytical model and shows possible exploitations of the paper results.

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